A Level Set/Vortex Sheet Method for Modeling Phase Interface Dynamics During Primary Breakup

M. Herrmann*
Center for Turbulence Research
Stanford University, Stanford, CA 94305

Abstract
The atomization of turbulent liquid jets and sheets can usually be divided into two subsequent processes: the primary breakup, where the jet or sheet initially breaks up into both large and small scale structures, followed by the secondary breakup, where these structures continue to break up into ever smaller drops, forming a spray. Considering numerical grid resolutions typical for Large Eddy Simulations (LES), the large scale liquid structures during primary breakup are well resolved, the liquid volume fraction can be of order one, and the phase interface can have any arbitrary complex shape. To correctly account for these characteristics, the phase interface has to be explicitly tracked on the numerically resolved scale and all phase interface dynamics on the subgrid scales have to be modeled. To derive such a Large Surface Structure (LSS) model, a level set/vortex sheet method is proposed. The level set/vortex sheet method tracks the phase interface by a level set scalar, so that topological changes of the interface, like breaking and merging, are handled automatically. Assuming inviscid fluids, the dynamics of the phase interface can be described by a vortex sheet located at the phase interface. The evolution equation for the vortex sheet strength then contains explicit local source terms for the physical processes at the phase interface, namely stretching terms, a surface tension term, and terms accounting for the differences in fluid density. The proposed level set/vortex sheet method thus provides a framework for the derivation of the primary breakup LSS subgrid models. Some preliminary results, namely the Kelvin-Helmholtz instability in the linear regime, the oscillations of liquid columns and spheres, and the three-dimensional breakup of a liquid/gas surface and a liquid sheet are presented in this paper.

Introduction
Atomization processes play an important role in a wide variety of technical applications and natural phenomena, ranging from inkjet printers, gas turbines, direct injection IC-engines, and cryogenic rocket engines to ocean wave breaking and hydrothermal features. The atomization process of liquid jets and sheets is usually divided into two consecutive steps: the primary and the secondary breakup. During primary breakup, the liquid jet or sheet exhibits large scale coherent structures that interact with the gas phase and break up into both large and small scale drops. During secondary breakup, these drops break up into ever smaller drops that finally may evaporate.

Usually, the atomization process occurs in a turbulent environment, involving a wide range of time and length scales. Given today’s computational resources, the direct numerical simulation (DNS) of the turbulent breakup process as a whole, resolving all physical processes, is impossible, except for some very simple configurations. Instead, models describing the physics of the atomization process have to be employed. Various models have already been developed for the secondary breakup process. There, it can be assumed that the characteristic length scale $\ell$ of the drops is much smaller than the available grid resolution $\Delta x$ and that the liquid volume fraction in each grid cell $\Theta_l$ is small, see Fig. ???. Furthermore, assuming simple geometrical shapes of the individ-

*Corresponding Author
ual drops, like spheres or ellipsoids, the interaction between these drops and the surrounding fluid can be taken into account. Statistical models describing the secondary breakup process in turbulent environments can thus be derived [?, ?, ?, ?, ?].

However, the above assumptions do not hold true for the primary breakup process. Here, the turbulent liquid fluid interacts with the surrounding turbulent gas-phase on scales larger than $\Delta x$, resulting in highly complex interface dynamics and individual grid cells that can be fully immersed in the liquid phase, compare Fig. ?? . An explicit treatment of the phase interface and its dynamics is therefore required. To this end, we propose to follow in essence a Large Eddy Simulation (LES) type approach: all interface dynamics and physical processes occurring on scales larger than the available grid resolution $\Delta x$ shall be fully resolved and all dynamics and processes occurring on subgrid scales shall be modeled. The resulting approach is called Large Surface Structure (LSS) model.

In order to develop such a LSS model for the turbulent primary breakup process, one potential approach is to start off from a fully resolved description of the interface dynamics using the Navier-Stokes equations and include an additional source term of the interface dynamics using the Navier-Stokes approach is to start off from a fully resolved description. The Level Set/Vortex Sheet method is presented. Then, introducing ensemble averaging or spatial filtering results in unclosed terms that require modeling [?, ?, ?, ?]. Unfortunately, the derivation of such closure models is not straightforward and, hence, has not been achieved yet. This is in part due to the fact that, with the exception of the surface tension term, all other physical processes occurring at the phase interface itself, like for example stretching, are not described by explicit source terms. Instead, they are hidden within the interdependence between the Navier-Stokes equations and the respective interface tracking equation. Thus, a formulation containing the source terms explicitly could greatly facilitate any attempt to derive the appropriate closure models.

To this end, a novel three-dimensional Eulerian level set/vortex sheet method is proposed. Its advantage is the fact that it contains explicit source terms for each individual physical process that occurs at the phase interface. It thus constitutes a promising framework for the derivation of the LSS subgrid closure models.

This paper is divided into four parts. First, the underlying governing equations of the level set/vortex sheet method for three-dimensional two-phase interface dynamics are presented. Second, the numerical methods employed to solve the level set/vortex sheet method are discussed. Then, simulation results for the growth of the Kelvin-Helmholtz instability in the linear regime and the oscillation of liquid columns and spheres are compared to theory in order to validate the proposed method. Then, preliminary three-dimensional results, i.e. the breakup of a randomly perturbed liquid surface and sheet are presented. Finally, conclusions are drawn and an outlook is given.

The Level Set/Vortex Sheet method

The objective of the level set/vortex sheet method is to describe the dynamics of the phase interface $\Gamma$ between two inviscid, incompressible fluids 1 and 2, as shown in Fig. ?? . In this case, the velocity $u_i$ on either side 1 or the interface $\Gamma$ is determined by the incompressible Euler equations, given here in dimensionless form,

$$ \nabla \cdot u_i = 0, \quad (1) $$

$$ \frac{\partial u_i}{\partial t} + (u_i \cdot \nabla) u_i = -\frac{1}{\rho_i} \nabla p, \quad (2) $$

subjected to the boundary conditions at the interface $\Gamma$,

$$ [(u_1 - u_2) \cdot n]_\Gamma = 0 \quad (3) $$

$$ \left[ n \times (u_2 - u_1) \right]_\Gamma = \eta \quad (4) $$

$$ \left[ p_2 - p_1 \right]_\Gamma = \frac{1}{\text{We}} \kappa \quad (5) $$

and at infinity,

$$ \lim_{y \rightarrow \pm \infty} u_i = \pm u_\infty. \quad (6) $$

Here, $n$ is the interface normal vector, $\eta$ is the vortex sheet strength, and $\kappa$ is the local curvature of $\Gamma$. The Weber number is defined as

$$ \text{We} = \rho_{\text{ref}} u_{\text{ref}}^2 / \sigma L_{\text{ref}}, \quad (7) $$

where $\sigma$ is the surface tension coefficient and $\rho_{\text{ref}}$, $u_{\text{ref}}$, and $L_{\text{ref}}$ are the reference density, velocity, and length, respectively. An interface subjected to the above boundary conditions is called a vortex sheet [?].

The partial differential equation describing the evolution of the vortex sheet strength $\eta$ can be derived by combining the Euler equations, Eqs. (??)
\[ \frac{\partial \eta}{\partial t} + \mathbf{u} \cdot \nabla \eta = -\mathbf{n} \times \left[ (\eta \times \mathbf{n}) \cdot \nabla \mathbf{u} \right] + n \left[ (\nabla \mathbf{u} \cdot \mathbf{n}) \cdot \eta \right] + \frac{2(A + 1)}{\text{We}} \left( \mathbf{n} \times \nabla \kappa \right) + 2A \mathbf{n} \times \mathbf{a}. \] (8)

Here, \( A = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) \) is the Atwood number and \( \mathbf{a} \) is the average acceleration of fluid 1 and fluid 2 at the interface. The major advantage of Eq. (8), as compared to a formulation based on the Euler equations, is the fact that Eq. (8) contains explicit local individual source terms on the right-hand side describing the physical processes at the interface. These are, from left to right, two stretching terms \( \mathbf{S} \), a surface tension term \( \mathbf{T}_\sigma \), and a density difference term.

In addition to the evolution of the local vortex sheet strength, Eq. (8), the location and motion of the phase interface itself has to be known. To this end, vortex sheets are typically solved by a boundary integral method within a Lagrangian framework where the phase interface is tracked by marker particles [?]. Marker particles allow for highly accurate tracking of the phase interface motion in a DNS. However, the introduction of ensemble averaging and spatial filtering of the interface topology is not straightforward and hence a strategy for the derivation of appropriate LSS subgrid closure models is not directly apparent.

Level sets, on the other hand, have been successfully applied to the derivation of closure models in the field of premixed turbulent combustion [?]. Thus, instead of using marker particles to describe the location and motion of the phase interface, here, the interface is represented by an iso-surface of the level set scalar field \( G(x,t) \), as shown in Fig. ??.

\[ G(x,t)|_\Gamma = G_0 = \text{const}, \] (9)

\( G(x,t) > G_0 \) in fluid 1, and \( G(x,t) < G_0 \) in fluid 2, an evolution equation for the scalar \( G \) can be derived by simply differentiating Eq. (9) with respect to time,

\[ \frac{\partial G}{\partial t} + \mathbf{u} \cdot \nabla G = 0. \] (10)

This equation is called the level set equation [?]. Using the level set scalar, geometrical properties of the interface, like its normal vector and curvature, can be easily expressed as

\[ \mathbf{n} = \frac{\nabla G}{|\nabla G|}, \quad \kappa = \nabla \cdot \mathbf{n}. \] (11)

Strictly speaking, Eqs. (8) and (9) are valid only at the location of the interface itself. However, to facilitate the numerical solution of both equations in the whole computational domain, \( \eta \) is re-distributed by setting it constant in the interface normal direction,

\[ \nabla \eta \cdot \nabla G = 0, \] (12)

and \( G \) is reinitialized by defining it to be a distance function away from the interface,

\[ |\nabla G|\big|_{G \neq G_0} = 1. \] (13)

Equations (8) and (9) are coupled by the self-induced velocity \( \mathbf{u} \) of the vortex sheet. To calculate \( \mathbf{u} \), the vector potential \( \psi \) is introduced,

\[ \Delta \psi = \omega. \] (14)

Here, the vorticity vector \( \omega \) is calculated following a vortex-in-cell type approach [?],

\[ \omega(x) = \int_V \eta(x') \delta(x-x') \delta(G(x') - G_0) |\nabla G(x')| \, dx', \] (15)

where \( \delta \) is the delta-function. Then, \( \mathbf{u} \) can be calculated from

\[ \mathbf{u}(x) = \int_V \delta(x-x') (\nabla \times \psi) \, dx'. \] (16)

In summary, Eqs. (8) and (9) together with Eqs. (10) - (13) describe the three-dimensional two-phase interface dynamics and constitute the level set/vortex sheet method.
Numerical Methods

The system of equations describing the phase interface dynamics is solved on an equidistant, Cartesian grid. However, instead of solving the equations throughout the whole computational domain, a tube approach is employed to significantly speed up the computations. Following arguments by Peng et al. [7], five distinct tubes around the \( G = G_0 \) level set, called \( I-, I+, B-, T-, \) and \( N- \) tube are introduced. The condition for a grid node \((i,j)\) to belong to one or more specific tubes is given by

\[
\begin{cases}
I & \text{if } |G_{i,j}| \leq \alpha_I \Delta x \\
I_2 & \text{if } |G_{i,j}| \leq 2\alpha_I \Delta x \\
B & \text{if } |G_{i,j}| \leq \alpha_B \Delta x \\
T & \text{if } |G_{i,j}| \leq \alpha_T \Delta x \\
N & \text{if } (i_0,j_0) \in T \text{ with } i_0 \in [i-3,i+3], j_0 \in [j-3,j+3],
\end{cases}
\]

with \( \Delta x \) being the grid size and typically \( \alpha_I = 4, \alpha_B = \alpha_I + 3, \) and \( \alpha_T = \alpha_B + 3 \). The use of the respective tubes will be discussed in the following sections.

Convective Terms

The level set equation, Eq. (17), is a Hamilton-Jacobi equation. In this work, a third order WENO scheme for Hamilton-Jacobi equations [?] is used. A Roe flux with local Lax-Friedrichs entropy correction (Roe-LLF) [?,?] is employed to solve both the level set equation, Eq. (17), and the convective term of the \( \eta \)-equation, Eq. (24). Integration in time is performed by a third order TVD Runge-Kutta time discretization [?].

Solution of the convective terms is restricted to the \( T- \) tube, where \( \mathbf{u} \) in Eqs. (18) and (19) is replaced by \[\mathbf{u}_{\text{cut}} = c(G) \mathbf{u},\] as well as the surface tension term \( T_\sigma \) in the \( \eta \)-equation, Eq. (24), are integrated in time within the convective TVD Runge-Kutta scheme. Since the source terms have physical meaning only at the interface itself, their evaluation is limited to the \( I- \) tube. For numerical reasons, these values are then redistributed to the \( N- \) tube by solving

\[
\nabla \mathbf{S} \cdot \nabla G = 0, \quad \nabla T_\sigma \cdot \nabla G = 0.
\]

Reinitialization

The reinitialization procedure employed here to solve Eq. (17) has originally been proposed by Sussman et al. [?], where the Hamilton-Jacobi type equation

\[
\frac{\partial H(x,t^*)}{\partial t^*} + S(H(x,t^*)) (|\nabla H(x,t^*)| - 1) = 0
\]

(21)

\[
H(x,t^*) = 0 = G(x,t) - G_0
\]

(22)

is solved for all \((i,j) \in N\) until

\[
||H(x,t^*) - H(x_0,t^* - \Delta t^*)||_\infty < \epsilon_1, (i,j) \in B
\]

(23)

with typically \( \epsilon_1 = 10^{-3} \Delta x \). In Eq. (23), \( S(H) \) is an approximation to the sign function. The proper choice of this approximation is crucial to minimize undesirable movement of the \( H(x,t^*) = 0 \) interface [?] while solving Eq. (23). Here, we will use the approximated sign function [?],

\[
S(H) = \frac{H}{\sqrt{H^2 + |\nabla H|^2(\Delta x)^2}}
\]

(24)

with a second order central difference approximation for \( \nabla H \). Equation (24) is solved by the third order WENO scheme but employing a Godunov instead of a Roe-LLF flux function. Again, the solution in time is advanced by a third order TVD Runge-Kutta scheme.

Redistribution

A Fast Marching method is employed to solve the redistribution equations (23) and (25). Here, we use the standard second order method as described in detail in [?,?]. Redistribution is performed in the \( N- \) tube only.

Employing a Hamilton-Jacobi equation based redistribution as proposed in [?] instead of the Fast Marching method proved to introduce too much undesired tangential transport in the redistributed scalars, \( \eta, \mathbf{S}, \) and \( T_\sigma \). Hence, this method has not been used here.
We = 10
0 10 100 1000 o

Figure 3. Growth rates $w$ of the Kelvin-Helmholtz instability in the linear regime, level set/vortex sheet method (●) and linear theory (line).

Velocity Calculation Methods

In order to solve the Poisson equations for the vector potential, Eq. (?), the vorticity at each grid node in the computational domain has to be calculated by a numerical version of Eq. (??). Approximating the delta function by a smoothed version, the vorticity, theoretically located solely on the interface, is in effect spread out onto the neighboring grid nodes, thereby prescribing a constant, non-zero local shear layer thickness. Hence, this approach is similar to the vortex-in-cell method that spreads the vorticity of Lagrangian vortex particles to their surrounding grid nodes [?, ?]. Quite a number of spreading functions have been proposed to this end [?]. Here, we will use

$$
\delta(x - x(s)) = \delta_\epsilon(x - x(s))\delta_\epsilon(y - y(s))
$$

$$
\delta(G(x') - G_0) = \delta_\epsilon(G(x') - G_0)
$$

with the numerical delta function $\delta_\epsilon$ as proposed by Peskin [?]

$$
\delta_\epsilon(x) = \begin{cases} 
\frac{1}{2\epsilon} \left[ 1 + \cos \left( \frac{\pi x}{\epsilon} \right) \right] & : |x| \leq \epsilon \\
0 & : |x| > \epsilon
\end{cases}
$$

and the spreading parameter $\epsilon$ set to $\epsilon = \alpha_I\Delta x$. The integration in Eq. (?) is performed by first evaluating

$$
\Omega(x', t) = \eta(x', t)\delta_\epsilon(G(x') - G_0) |\nabla G(x')|
$$

for all cells within the $T$-tube and then integrating

$$
\omega(x, t) = \int_{V'} \delta(x - x')\Omega(x', t)dx'.
$$

for all cells within the $T_2$-tube by a simple midpoint rule, with $V'$ the area, where $|x - x'| \leq \epsilon$, $|y - y'| \leq \epsilon$, and $|z - z'| \leq \epsilon$.

The Poisson equations for the vector potential, Eq. (?), are solved by the package FISHPACK [?] throughout the whole computational domain employing appropriate boundary conditions.

Finally, the calculation of the vortex sheet-induced velocity $u$ from the vector potential, Eq. (?), is again a two-step process. First, an initial velocity $U$ is calculated at each grid node by second order central differences of $\psi$. Second, an interpolation step using the same numerical integration method and $\delta$-function has to be employed,

$$
u(x, t) = \int_{V'} \delta(x - x')U(x', t)dx',
$$

evaluated within the $T$-tube. Note that strictly speaking, this interpolation step is not fully consistent with the spreading step, Eqs. (?) and (?), because a matching $\delta(G)$ term in Eq. (?) cannot be defined in Eq. (?).

Results

In order to both validate the proposed level set/vortex sheet method and to demonstrate its ability to perform DNS of the primary breakup process, the results of three different test cases are presented. First, the level set/vortex sheet method is used to calculate the growth rates of the two-dimensional Kelvin-Helmholtz instability in the linear regime.

Figure 4. Oscillation period $T$ of liquid columns (left) and spheres (right) as a function of mode number $n$ for varying Weber numbers $We$, level set/vortex sheet method (●) and linear theory (line).
spheres are calculated and compared to theoretical results. Finally, the temporal evolution of a randomly perturbed three-dimensional surface and sheet are presented.

**Kelvin-Helmholtz Instability in the Linear Regime**

The objective of this test problem is to validate the level set/vortex sheet method by comparing the calculated growth rates of the Kelvin-Helmholtz instability in the linear regime to those obtained by linear theory. The initial vortex sheet strength distribution is calculated from

\[ \eta(x, t = 0) = \frac{w(\text{We})}{w(\text{We} = \infty)} (\eta_{\text{VS}}(x, t = 0) - \eta^*) + \eta^*, \]

where \( w(\text{We}) \) is the growth rate predicted by linear theory \([2], \eta^* = -1\), and \( \eta_{\text{VS}} \) is given by

\[ \eta_{\text{VS}}(x, t = 0) = \eta^* \sqrt{1 + \frac{4\pi A_0}{B} \cos \frac{2\pi}{B} x + 2 \left[ \frac{2\pi A_0}{B} \cos \frac{2\pi}{B} x \right]^2}. \]

Periodic boundary conditions are employed at the left and right domain boundary. The velocity at the upper and lower boundary is set to

\[ u(x, y = \pm 0.5) = (\pm 0.5, 0)^T, \]

respectively. The simulations are performed in a \( B \times B \) box on an equidistant \( 256 \times 256 \) Cartesian grid. The spreading parameter is set to \( \varepsilon = 4/256 \).

Figure 5 compares the calculated growth rates \( w \),

\[ w = \frac{1}{t_1} \int_0^{t_1} w(t) dt, \quad t_1 = 0.5. \]

Although the level set/vortex sheet method marginally over-predicts the linear growth rate, agreement between simulation results and linear theory is very good. The reason for the slight over-prediction is not directly apparent, but it is most likely due to the lack of a consistent level set based interpolation step to calculate the vortex sheet induced velocity \( u \), Eq. (32).

Figure ?? shows the distribution of the stretching term \( S \) and the tension term \( T_\sigma \), along the normalized arc length \( s/L \) at \( t = 0 \). For \( \text{We} = 13 \), both terms almost balance, resulting in a small growth rate \( w \). For increasing \( \text{We} \) numbers, the stretching term \( S \) starts to dominate the surface tension term \( T \), resulting in larger growth rates \( w \). For \( \text{We} = \infty \), secondary instabilities of higher wave numbers emerge in the distribution of the stretching term. This is consistent with linear theory, since the Kelvin-Helmholtz instability is an ill-posed problem in the absence of surface tension forces. The formation of the Moore singularity \([2]\) in the limit of \( \text{We} \to \infty \) is avoided here, because the level set/vortex sheet method introduces two types of desingularization into the governing equations. For one, an underlying numerical grid is employed, thereby limiting the numerical evaluation of any gradients. Furthermore and more importantly, the spreading parameter \( \varepsilon \) introduces a constant, non-zero shear layer thickness, thereby mimicking the desingularization effects of viscosity \([2]\). Still, in the limit of \( \text{We} \to \infty \), disturbances of higher wave number caused by numerical errors have significantly higher growth rates than the initial disturbance, thereby becoming dominating for later times.

**Oscillating Columns and Spheres**

To further validate the proposed level set/vortex sheet method, the calculated oscillation periods \( T \) of liquid columns and spheres of mean radius \( R = 0.25 \), center \( x_c = (0.5, 0.5, 0.5) \), amplitude \( A_0 = 0.05 R \), and Atwood number \( A = 0 \) are compared to theoretical results \([2]\). The initial vortex sheet strength
in both cases is set to
\[ \eta(x, t = 0) = 0. \]  
(36)

All calculations are performed in a unit sized box resolved by an equidistant Cartesian grid of $128 \times 128$ and $128 \times 128 \times 128$ nodes, respectively.

Figure 6 depicts on the left the oscillation period $T$ of liquid columns for varying mode numbers $n$ and two different Weber numbers. The corresponding results for the oscillating spheres are shown on the right. As can be clearly seen, agreement between simulation and theory is very good.

Figure ? depicts the distribution of the surface tension term $T_{\sigma}$ of the $\eta$-equation (36) in the $x$-, $y$-, and $z$-direction, $T_{\sigma} = 2(A + 1) \frac{\eta}{\text{We}} (n \times \nabla \kappa)$, for the oscillating sphere with $n = 5$ and $\text{We} = 10$ at $t = 0$. As the shape of the sphere indicates, $T_x$ in the $x$-direction is a factor of roughly four higher than $T_{\sigma}$ in the other two directions, leading to the predominant oscillation in the $y$-$z$-plane.

**Liquid Surface and Sheet Breakup**

To demonstrate the capability of the proposed level set/vortex sheet method to simulate the primary breakup process, the temporal evolution of both a randomly perturbed liquid surface and sheet are simulated. In the case of the liquid surface, the on average flat interface located at $z = 0$ is perturbed in the $z$-direction by a Fourier series of 64 sinusoidal waves. The thickness of the liquid sheet is set to $C = 0.1$.

The initial vortex sheet strength for the liquid surface is set to
\[ \eta(x, t = 0) = (-1, 0, 0)^T \]  
(38)

and to
\[ \eta(x, t = 0) = \begin{cases} (-1, 0, 0)^T & : z > 0 \\ (1, 0, 0)^T & : z \leq 0 \end{cases} \]  
(39)

in the liquid sheet case. Both, the surface and the sheet simulation were performed in a $x$- and $y$-direction periodic box of size $[0, 1] \times [0, 1] \times [-1, 1]$ resolved by a Cartesian grid of $64 \times 64 \times 128$ equidistant nodes. In both simulations, the Atwood number is $A = 0$. The Weber number in the surface simulation is $\text{We} = 500$ and the Weber number in the sheet simulation based on the sheet thickness is $\text{We}_C = 100$.

As depicted in Fig. ??, the liquid surface shows an initial growth of two-dimensional Kelvin-Helmholtz instabilities $(t = 1)$. These continue to grow $(t = 3)$ and form three-dimensional structures $(t = 5)$ resulting in elongated fingers $(t = 6.5)$ that finally initiate breakup $(t = 8.0)$.

The liquid sheet, depicted in Fig. ??, again exhibits the initial formation of two-dimensional Kelvin-Helmholtz instabilities $(t = 1)$ that continue to grow $(t = 3)$ until the liquid film gets too thin and ruptures $(t = 5)$. Individual fingers are formed that extend mostly in the transverse direction $(t = 8)$ and continue to break up into individual drops of varying sizes $(t = 12)$.

**Conclusions and Outlook**

A Eulerian level set/vortex sheet method has been presented that allows for the three-dimensional
Figure 7. Temporal evolution of the three-dimensional liquid surface breakup, $A = 0$, We = 500.
Figure 8. Temporal evolution of the three-dimensional liquid sheet breakup, $A = 0$, $We_C = 100$. 
calculation of the phase interface dynamics between two inviscid and incompressible fluids. Results obtained with the proposed method for the growth of the Kelvin-Helmholtz instability in the linear regime and for the oscillation periods of liquid columns and spheres show very good agreement with theoretical predictions. Furthermore, the applicability of the method to the primary breakup process has been demonstrated by simulations of the breakup of both a liquid surface and a liquid sheet.

The proposed level set/vortex sheet method has the advantage that it allows for the detailed study of each individual physical process occurring at the phase interface, because they appear as explicit local source terms in the governing equations. Coupling of the level set/vortex sheet method to an outside turbulent flow field will allow for DNS of the primary breakup process to help identify characteristic regimes of turbulent primary breakup and their dominant physical processes. The level set/vortex sheet method thus provides a promising framework for the derivation of LSS subgrid models.

Acknowledgments

The support of the German Research Foundation (DFG) is gratefully acknowledged.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Atwood number</td>
</tr>
<tr>
<td>$A_0$</td>
<td>initial amplitude</td>
</tr>
<tr>
<td>$a$</td>
<td>average acceleration</td>
</tr>
<tr>
<td>$B$</td>
<td>size of the computational domain</td>
</tr>
<tr>
<td>$C$</td>
<td>liquid sheet thickness</td>
</tr>
<tr>
<td>$G$</td>
<td>level set scalar</td>
</tr>
<tr>
<td>$\ell$</td>
<td>integral length scale</td>
</tr>
<tr>
<td>$n$</td>
<td>mode number</td>
</tr>
<tr>
<td>$\mathbf{n}$</td>
<td>normal vector</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
</tr>
<tr>
<td>$S$</td>
<td>stretching terms</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T_S$</td>
<td>surface tension term</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>grid size</td>
</tr>
<tr>
<td>$w$</td>
<td>growth rate</td>
</tr>
<tr>
<td>We</td>
<td>Weber number</td>
</tr>
<tr>
<td>$\delta$</td>
<td>delta function</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>spreading parameter</td>
</tr>
<tr>
<td>$\eta$</td>
<td>vortex sheet strength</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>curvature</td>
</tr>
<tr>
<td>$\psi$</td>
<td>vector potential</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>surface tension coefficient</td>
</tr>
<tr>
<td>$\Theta_l$</td>
<td>cell liquid volume fraction</td>
</tr>
<tr>
<td>$\omega$</td>
<td>vorticity vector</td>
</tr>
</tbody>
</table>

Subscripts
- ref: reference quantities

Superscripts
- $T$: transpose

References


